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MASTER´S FINAL WORK

DISSERTATION

The application of Benford's Law in detecting accounting fraud in the Financial Sector

AMÉLIA SOFIA CARVALHO DAMIÃO DA SILVA

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SUPERVISION:

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Abstract

This study aims to apply Benford's Law in detecting accounting fraud on the analysis of net income from listed financial companies, through years 2003-2012. Its main purpose is to confirm whether Benford’s Law is still valid after the 2007/2008 financial crisis. To measure the statistical significance, a Z test with a 95% confidence interval was employed. With the exception of a sample showing positive net income after 2008, for the digits 1, 6 and 9, it was confirmed that our sample follows Benford's Law which demonstrates the effect of the 2007/2008 crises on financial companies.

Keywords: Benford's Law, Net Income, Accounting Fraud, *Subprime*

Resumo

Este estudo pretende aplicar a Lei de Benford na deteção de fraude contabilística, através da análise dos resultados líquidos das empresas financeiras cotadas durante os anos de 2003 a 2012. O principal objetivo é verificar se esta lei se mantém após a crise de 2007/2008. Como medida de significância estatística utilizou-se o teste Z com um intervalo de confiança de 95%. Confirma-se que a nossa amostra segue a lei de Benford à exceção dos resultados líquidos positivos após o ano 2008 para os dígitos 1, 6 e 9,o que demonstra o efeito da crise de 2007/2008 nas empresas financeiras.

Palavras Chave: Lei de Benford, Resultado Liquido, Fraude Contabilística, *Subprime*

Acknowledgments

First of all, I would like to express my heartfelt gratitude to professor Dr. Maria João Coelho Guedes, my thesis adviser, for her always available support proposal for improvement and the given knowledge. She consistently allowed this paper to be my own work, but steered in the right direction whenever she thought I need it.

I want to thank also to my parents Mário and Rosa, sister Sílvia and brother Mário Miguel, for the support, comprehension and suggestions.

I wish to convey special thanks to Marco, my boyfriend for the given support and incentive.

Finally, I could not miss this opportunity to thank to my good friends Tânia Damião and Tânia Patrícia for always remembering me that I should finish my thesis on time.

List of Abbreviations

ASB - Portuguese Bank Association

ATM- Automated Teller Machine

BD- Benford's Distribution

BL- Benford's Law

BPD- Benford's Probability Distribution

CCE Cash and Cash Equivalents

CI- Credit Institution

EBIT- Earnings before Interest and Taxes

EBTIDA- Earnings Before Taxes Interest Depreciations and Amortizations

EC- Equity Capital

EF- Expected Frequency

EPS- Earnings Per Share

EU- European Union

FS- Financial Sector

GFCIFC- General Framework for Credit Institutions and Financial Companies

IQ- Intelligence Quotient

IS- Income Statement

ISA – International Standard on Auditing

K- 1.000 Units

NI- Net Income

NYSE – New York Stock Exchange

Od- Observed Distribution

OF- Observed Frequency

Td- Targeted Distribution

USA- United States of America

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I. Introduction

Financial fraud thematic has become more and more relevant through time. It's been attracting a great deal of concern and attention, thus becoming a global issue, due to consecutive scandals such as in the US (World.com, Enron,Lehman Brothers), Italy (Parmalat), France (Vivendi) and Portugal (BPN).

The accounting system is one of the main targets of fraud abuse, and there are several incentives leading to this act, such as an individual personal usufruct of money, manipulation of financial information in order to elude *stockholders* and altering financial fgures with the purpose of achieving awards. A clear example of the latter is the indexed salary schedules.

An early detection of fraud is vital to prevent its devastating consequences and implications affecting investors, regulators, auditors and general public.

Benford's Law (BL) was empirically formulated in the XIX century and, lately, it has been applied to detect accounting fraud. So, we are going to apply this same law to the net income (NI) of financial companies in several world indexes, from 2003 to 2012 and establish whether they follow the same distribution. Likewise, we will make a split of analysis between the period before and after 2008, the year of the great financial crises. This way, we'll be able to assess the validity of our sample data; however, we are not going to draw conclusions on the validity of the NI from a particular entity.

The results obtained allowed us to confirm that our sample follows BD on the NI reported during 2003-2012, with the exception of positive NI after the year 2008.

The remainder of this dissertation is structured as follows. The second chapter corresponds to the literary revision. There, some key concept are presented: the definition of financial fraud, Benford's law, the empirical application of Benford's law, its application to the detection of accounting fraud and it's application to the companies' net income. Data definition, variables in study, sectors in study and methodology are presented in the third chapter. The results' analysis corresponds to the fourth chapter; and, finally, the fifth chapter provides a conclusion with final considerations, criticisms, limitations of the study and suggestions for future inquiries.

II. Literary Review

2.1 Financial Fraud – Conceptualization

According to Wang et *al*. (2006) fraud is a deliberate act that goes against a law, rule or policy with the intention of obtaining an unauthorized financial benefit. According to (Simmons, 1995) fraud occurs when the following elements exist:

- An individual or an organization, deliberately makes an untrue representation about an important fact or event;
- The victim believes the untrue representation;
- The victim bases itself on (and acts accordingly to) that untrue representation;

- The victim suffers loss of money or estates due to acting in accordance to the untrue representation.

There are several sorts of fraud like bribery, political corruption, workers' fraud on financial statements (Albrecht *et al.*, 2008).

In this dissertation, we are going to focus on fraud related to financial statements. This type of fraud involves schemes implicating the company's name and generally is carried out by altering its financial statements or by inappropriate disclosures, in order to improve its image (Wells, 2002).

In the ISA 240 (International *Standard on Auditing*) this is defined as comprising all deliberate distortions of financial statements. The ISA also mentions auditor's responsibilities relating to fraud, being that he is responsible for assuring that the financial statements are free from materially prominent errors or distortions, which can be caused by fraud or error.

According to Abbasi *et al.*(2012) this kind of fraud can have several consequences to the sustainability of an organization, as well as adverse effects on its staff members, investors and capital market throughout the world, thus provoking the loss of confidence and integrity in the business world. (Albrecht *et al.*, 2008).

These consequences were observed in consecutive scandals, such as the one involving the company Enron Corporation (Texas, USA) , where a fictitious appreciation of assets occurred, in order to present better financial statements, as a way of increasing stockholders confidence. This plan was in such a way well designed, that neither the auditors of Arthur Anderson, neither the risk evaluation companies were able to identify the fraud. The known WorldCom's scandal case when, in June of 2002, it was revealed that $3,8 billion costs had been inappropriately hidden by means of considering an expense as an asset, which lead to an alteration on the financial situation of the company. (Collins, *et al.*, 2005)

These types of fraud contributed to the bankruptcy of the biggest world's largest companies. As it can be seen on table I, four of the biggest US bankruptcies happened due to fraud. (Abbasi *et al.* 2012).

Table I-Largest Bankruptcy data in U.S History

Company	**Assets (Billions)**	**When Filed**	**Fraud Involved?**
1. Lehman Brothers, Inc.	**$691,0**	**September 2008**	**Yes**
2. Washington Mutual,Inc.	$327.9	September 2008	Not Yet Determined
3. WorldCom, Inc.	**$103.9**	**July 2002**	**Yes**
4. General Motors Corp.	$91.0	June 2009	Not Yet Determined
5. CIT Group, Inc	$80.4	November 2009	Not Yet Determined
6. Enron Corp.	**$65.5**	**December 2001**	**Yes**
7. Consenco, Inc.	**$61.4**	**December 2002**	**Yes**
8. Chrysler, LLC	$39.3	April 2009	Not Yet Determined
9. Thornburg Mortage, Inc	$ 36.5	May 2009	Not Yet Determined
10 Pacific Gas & Electric Co.	$36.2	April 2001	No

Source: Bankruptcydata. Com (2010)

A falsified financial statement affects the economy of all countries, especially when involving great companies, such as Enron and World.com. According to Cotton (2002) and Rezaee (2002), the cost resulting from these frauds was over 500$ billion, in recent years.

It is essential to present true statements, so that investors can decide adequately about the opportunities and risks concerning investment alternatives. In order to know about a company's past performance, its current "health" and perspectives for the future, financial statements are the best source of information (Collins *et al.*, 2005).

Therefore, financial information has been used to detect fraud by several authors, such as Cecchini *et al*. (2005), Kirkos *et al*. (2007) and Green & Choi (1997).

2.2 Benford's Law

2.2.1 Benford's Law Definition

Newcomb-Benford's Law was empirically discovered by the end of the XIX century. Its story began in 1881, with a discovery made by an American astronomer and mathematician. While flicking through much used logarithmic tables on the library, Simon Newcomb noticed that the front pages were much more eroded that the last ones, which lead him to the conclusion that digits begun with the number 1 were much more frequent than the digits begun with the number 2 and so on. He thus concluded that the probability of any digit (N) being the first of any number equals to: $\log_{10}(N+1)-\log_{10}(N)$ (Newcomb, 1881).

The frequencies relating to the probability of a digit being the first one of a determined number can be found on table II:

Table II-Benford's Law frequency for the 1st digit

First Digit	***Probability***
1	30,10%
2	17,61%
3	12,49%
4	9,69%
5	7,92%
6	6,69%
7	5,80%
8	5,12%
9	4,58%

Source: Own elaboration.

We can be lead to think that, in a number, all digits from 1 to 9 have the same probability of occurring in the first position. However, contrarily to general thought, Benford's Law refers that, in the majority of real situations, as the first digit gets bigger the frequency diminishes. For this reason, this law also is called the First-Digit Law. As such, digits 1, 2 and 3 have bigger probability of appearing than digits from 4 to 9. The probability of the digit 1 appearing as a first digit is of around 30%, being that the one relating to the digit 9 barely reaches 5%, as can be seen on table II. That law can be quickly explained with a real example. If we invest 10€ (digit 1), our investment should grow 100% to reach 20€ (digit 2), but it needs only to grow 50% to reach 30€ (digit 3) and so on; each time becoming more and more difficult to change from digit 1 to 2 than from 2 to 3. That is why, in the real world, there's a bigger probability of 1 being the first digit of a determined number.

In 1938, Frank Benford arrived to the same conclusion, having extended the analysis of Newcomb to the 2nd and 3rd digits and to more than one number, with the following equation:

$$P(d) = \overset{10^{n-1}-1}{\underset{k=10^{n-2}}{\log 10}} (1 + (1/(10k + d))) \qquad (1)$$

Where:

d = A digit from 0 to 9

n = Digit's position (n> 1)

The frequencies of each digit, as were established by Benford, can be consulted in the following table III:

Table III- Expected Frequencies according to Benford's Law for the 1st to the 4th digit of a number

Digit	1st place	2nd place	3rd place	4th place
0		0.11968	0.10178	0.10018
1	0.30103	0.11389	0.10138	0.10014
2	0.17609	0.19882	0.10097	0.1001
3	0.12494	0.10433	0.10057	0.10006
4	0.09691	0.10031	0.10018	0.10002
5	0.07918	0.09668	0.09979	0.09998
6	0.06695	0.09337	0.0994	0.09994
7	0.05799	0.09035	0.09902	0.0999
8	0.05115	0.08757	0.09864	0.09986
9	0.04576	0.085	0.09827	0.09982

Source: (Nigrini, 2006)

After observing 20,000 different data, Benford concluded that the expected frequencies applied to several kinds of data, such as cities' population, length of the rivers and lakes and even the atomic weight of the elements (Benford,1938). According to Nigrini (2000), Benford's table was the most complete, available in the 90s.

In 1961, the mathematician Roger Pinkham studied what would happen to the distribution of Benford if it were multiplied by an arbitrary constant. For instance, if a determined numerical value in US Dollars (USD) is converted to Euros (EUR) by multiplying a constant, this multiplication does not alter the applicability of Benford's law (rule of invariance). Pietronero *et al.* (2001) and Raimi (1976) also concluded that this law complies with the scale and the condition of invariance.

Benford's Law (BL) has several applications, from detecting accounting fraud, fraud associated to electoral data, macroeconomic data (Nye & Moul, 2007), genetic facts (Friar JL, 2012) to even detecting scientific fraud.

Durtschi *et al.* (2004) concludes that this law, when properly used, is an important tool in case of fraud suspicion and analysis of accounting data.

This law can also be applied to population data. In 2002 Sandrom et al,. studied the application of this law to the distribution of population between countries, having confirmed that Benford’s Distribution (BD) is applied to territorial areas and population densities.

This law has some limitations, though, such as the fact that it cannot be applied to data where the sample is small; nor to numbers with minimums to the maximum (for instance, it cannot be applied to mobile numbers from Portugal, because they all start with 9); nor artificially generated numbers; nor numbers that have human influence; nor numbers that are constituted by just one or two digits, like human height or the results of an Intelligence Quotient (IQ) test (Fewster, 2009; Smith, 2012)

Other investigators have also been referring that this law is irrelevant to the evaluation of diverse facts. Decker (2011) mentions it is irrelevant to evaluate elections taking into account a good democratic practice and Thomas (1989) mentions that this law does not apply to the Earnings per Share (EPS) of a company.

2.2.2 Empirical application to data that follows Benford's Law

According to Krakar and Zgela (2009) data that follows Benford's distribution should have the following characteristics:

- They describe values relatively to the same/similar phenomenon (exemplifying, data relating to the height of mountains or the value of companies' sales);
- They should not have defined minimums or maximums;

- They cannot be attributed numbers , like in the case of telephone numbers or bank account numbers;
- They do not apply to numbers influenced by psychological factors. Several studies substantiate this last characteristic, as Hill (1988) concluded after analyzing 742 data generated by university students, that when people are requested to come up with "random numbers", these numbers do not follow Benford's distribution. Busta and Weinberg (1998) arrived to the same conclusion concerning computer generated numbers. The difficulty of fabricating numbers is related to psychological factors. Creating numbers can become difficult to individuals due to the fact that they consistently choose numerical sequences and avoid numbers repetition (Dubinsky, 2001).

According to Durtschi *et al.*(2004), the key for numbers to follow BL is to combine numbers from different sources, that is, to combine numbers that are not related so that they will be random (Hesman, 1999).

BL can be applied to distributions that follow mathematical functions. According to Berger, *et al.* (2005) the functions power, exponential and rational follow BL, at least for a great amount of initial numbers. Forman (2010), on the other hand, replicates the same study on more mathematical functions, like the exponential, uniform distribution and chi-square, being that the results met those of Berger *et al.* (2005).

Some accounting data follows BD because they are the result of a mathematical process, as was confirmed by Boyle in 1994 who concluded that when elements are random and originated from different sources that were multiplied, divided or multiplied in power, they follow BL (Durtschi *et al.*, 2004).

In 1995, Hill demonstrated that BL can be applied to the stock market, statistical census and accounting information (Durtschi *et al.*, 2004).

According to Raimi (1976), data that follows a geometrical distribution follow BL, for instance, the Fibonacci sequence (1,1,2,3,5), follows BD for a sufficiently big sample.

The characteristics of the distributions that do not follow BL are the following, according to Wendy and Brian (2007):

- Attributed numbers (e.g.: check number, invoice number);
- Numbers influenced by humans (e.g.: prices attributed by psychological limits);
- Accounts on which your data is influenced by specific characteristics of the company;
- Accounts with a minimum or maximum limit;
- Where no transaction is registered.

Nigrini and Mittermaier (1997) refer that some data do not follow BD, like signed numbers, numbers of checks or ATM withdrawals, supermarket prices and NYSE stock prices.

2.2.3 Benford's Law applied to Accounting Fraud

Several authors have been applying Benford's Law to ascertain the validity of accounting data, among them are Browne (1998), Carslaw (1988), Durtschi *et al.* (2004), Varian (1972) and Nigrini (1996).

In 1972, Hal Varian suggested that this law would be able to be used to detect possible frauds in socioeconomic data listings. He assumed that those individuals who fabricate numbers have a tendency to do so uniformly. Thus, by comparing between the first digit of the observed frequencies data and Benford's expected probabilities, one could detect deviant data that would indicate a deliberate error, that is to say, a possible fraud, or a non-deliberate error.

In the year of 1988, Carslaw carried out a study about the second digit present in the results of New Zealand's companies, with the objective of examining if their net result digits followed a random distribution. In his study, he concluded that the number 0 was the one with the biggest frequency, and that the number 9 was the one presenting a minor frequency. Afterwards, Thomas (1989) replicated Carslaw's study (1988) on North American companies and obtained the same result, which was that accounting data follows a Benford's distribution.

In the 1990s, there was an increment of inquiries about the application of this law in the detection of accounting fraud (Durtschi *et al*. 2004; Zgela & Dobsã, 2011; Forster, 2006; Hill, 1995, 1996; Nigrini 1996 a,b, 1997,2000)

The possibility that Benford's Law could be applied in detecting accounting fraud emerges with Nigrini who, in 1994, came up with the proposition that the information created by an individual is not random. Having been one of the investigators that most contributed to proof the applicability of this law in fraudulent plans - through the study of standards - Nigrini established that many accounting variables, like sales, purchases, price history and assets and passive accounts, follow Benford's distribution.

In 1996, Nigrini referred that this law was a secure approach and it could be used in fraud detection in a broad data sample, because it detects potential unreal numbers (Nigrini & Wood, 1996)

Still in 1996, Nigrini, pursues his study focusing in the first and second digit of paid and received value of contributions to the State, between 1985 and 1988. In his research, he confirmed that the examined data followed BD, concluding that contributors that paid less taxes had a higher tendency to fabricate numbers when declaring their earnings (Nigrini, 1996)

Browne (1998) referred that BL is a powerful tool to pinpoint fraud suspicions, fiscal evasion, misleading accounting and even errors in computer programs.

The demonstration of the applicability of BL in accounting fraud detection was carried out by Nigrini (1999). While studying a case of fraud in the state of Arizona, he analyzed a fraudulent payroll, noticing that data diverged significantly from BL, and that, in the last 5 reported years, these divergences were more significant, which coincided with the fact that higher numbers began to appear. In the same article the author refers that it is not possible to contrive a series of unreal numbers that conform to BD and so, manipulated numbers are incompatible with BL (Nigrini, 1999).

In his several analyses, Nigrini also mentions that the expected probabilities should be compared with the observed probabilities, being that significant deviations could suggest that the information is not reliable. (Nigrini, 1994,1996, 1998, 1999, 2000).

Durtschi *et al.* (2004) mentions limitations of this law in fraud detection, such as in the case of unregistered transactions, and other sorts of undetectable frauds, since its data does not follow a Benford's distribution, as in the case of bank accounts and invoice duplicates.

Thomas (1989) refers that BL is not applicable to the first, second and even the third digit of a company's EPS, since this variable follows a different standard. There are, he suggests, two reasons for this: 1) There's a great unexpected frequency of the numbers 5 and 0 in the 3rd digit, which indicates that this is the last digit and results from a rounding of numbers; 2) The EPS between -10 and 10 do not report the second digit.

2.2.4 Benford's Law applied to the companies' net income

The issue of the applicability of Benford's Law (BL) in the companies' net income (NIs) arose with Carslaw (1988) and Thomas (1989), who applied BL in NIs of companies from New Zealand and the United States of America; and confirmed that this variable followed BD. Thomas (1989), arrived to the conclusion that managers had a tendency for rounding off the companies' profit values, but did not do the same when registering its losses; which lead to an excessive occurrence of the digit 9 in the negative net income. So, the two distributions followed opposite directions.

Several authors state that companies tend to manipulate profits: firms that want their profits to appear bigger than they really are, tend to manage their digits upward, 1 or 5; and firms that want their profits to appear smaller tend to move their digits downwards, like 9 or 4. (Carslaw, 1988; Thomas, 1989; Skousen, *et al.*, 2004)

Other authors have applied BL in detecting profit fraud, such as Guan, *et al.,* (2006) that used BL to detect fraud in quarterly earnings reports of cosmetics firms, having observed that the less audited quarterly report is the one that shows less manipulation. Later, in 2008, Guan, *et al.* applied the same study for Thai and Japanese companies, obtaining even results.

Forster (2006), applied BL to 159 non-profitable organizations from the federal districts of Brazil, confirming that the majority of accounting data fit in Benford's distribution (BD).

Recently, several studies emerged regarding the applicability of this law, as is the case of Lin, *et al.*, (2011), who used BL to demonstrate that Thai companies tend to report profits with excessive occurrence of the numbers 5 and 10. That's because, as concluded by Zhou, (2010), numbers beginning with 5 tend to give a more optimistic answer in situations of low response rates in the stock market.

Zgela and Dobsã (2011) analyzed NIs of 500 companies, during the years of 2007, 2008 and 2009. They confirmed, once more, that the data was in conformity with BD.

Arcahambault & Arcahambault (2011), analyzed the profit from regulated and non-regulated Moody's companies, and concluded that both kinds tend to manipulate results, but in different ways.

III. Data Definition and Methodology

3.1 Data Definition

3.1.1 Sample Definition

The sample for this study comprises 717 companies of the financial sector, listed in different world indexes, during the years of 2003 to 2012. The number of companies by index is shown in table IV.

Table IV- Number of Sample Companies by World Index

Country/ Continent	Index	Number of Companies
Brazil	BRASIL IBOVESPA INDEX	5
France	CAC 40 INDEX index	1
Germany	DAX INDEX index	2
USA	DOW JONES AVG index	1
Europe	Euro Stox 50 Pr. Index	10
England	FTSE 100 INDEX index	13
China	HANG SENG INDEX Index	6
Spain	IBEX 35 INDEX index	6
USA	NASDAQ	478
Japan	NIKKEI 225 Index	21
America	S&P / asx 200 INDEX Index	22
Others	S&P 500 Financials (Sector)	72
China	SHANGAI SE COMPOSITE Index	25
Others	SMALLCAAP 2000	50
Switzerland	SWISS MARKET INDEX Index	5
-	**Grand Total**	**717**

Source: Bloomberg

The companies from the financial sector considered for this sample were: banks, insurance companies and financial services companies.

Since it will focus on positive and negative NI and, additionally, on the period before 2008 and on the one after 2008, our analysis is divided in the following ways, as you can be seen in table V:

Table V-Number of sample companies, considering positive and negative NI and time periods before and after 2008

Data Number	2007-2008	2008-20012
Negative NI	144	661
Positive NI	2643	2742

It is important to that the dimension of the company is not relevant, since our study will be based on the analysis of the NI's first digit; in other words, when considering a company where the NI value is 200K and another one where the NI is 20K, it will only be considered the first digit (with the exception of the zero to the left that doesn't count) which is the number 2 (Simikin, 2010).

The financial data of our sample is expressed in US dollars (USD), with values obtained from Bloomberg and already converted, using the current exchange rate, from the Original Currency to USD, since the our study is transversal to the different world stock markets, comprising many different currencies. The application of the exchange rate does not alter the applicability of BL, since the main feature of Benford's distribution is the rule of the invariance; that is to say, it can be multiplied by any constant without altering its effect. (Pinkham, 1961; Raimi, 1976; Pietronero *et al.*, 2001).

Our data sample fulfills the requirements of BL, that is: 1) it describes values within the same phenomenon; 2) it hasn't a defined minimum or maximum; 3) the values are not attributed numbers and 4) they are not influenced by psychological factors. Krakar and Zgela (2009).

According to Simikin (2010), for statistical purposes the sample should comprise over 100 observations, being that our sample also fulfills this requisite.

So, we will assess the validity of our data on a sector; this study, however, does not allow us to draw conclusions on the validity of a yearly NI from a particular entity.

3.1.2 Definition of the Sector in Study

This study aims to analyze the NI of financial sector entities, listed in different world stock markets, during the period of 2003 to 2012.

The financial sector was chosen due to its importance in the economy; due to the crisis of 2007/2008; and because it is one of the sectors where more fraudulent events occur. According to the annual report on fraud of 2012, the biggest victims of fraud are the bank industry and the financial sector with 16,7% (Association of Certified Fraud Examiners, 2012).

According to the Portuguese Bank Association (APB), the financial system encompasses the collective of financial institutions which assure, essentially, the flow of investment savings in the financial markets, by means of purchasing and selling financial products (Portuguese Bank Association, 2012).

The Portuguese General Framework for Credit Institutions and Financial Companies (GFCIFC) divides financial entities into two main groups; Financial Corporations (FC) and Credit Institutions (CI).

According to APB, the financial institutions (FI) play an intermediary role between economical agents which sometimes act as savers and other times as financial investors (Portuguese Bank Association, 2012). Generally speaking, the financial system has the purpose of transferring the investors' resources to the productive sector (Sullivan & Steven, 2003). Therefore, according to Allen & Douglas (2001), they play a crucial role in the allocation of resources in a modern economy.

In 2007, the *Subprime* crisis in the USA, led Europe and the rest of the world into a global crisis, without precedents since the 1930s. The banks' lack of mutual trust took the economy into a deep recession (Fassin & Gosselin, 2011)

According to Bessis (2010) the 2007/2008 crisis was due to the skepticism of financial companies, regulations, legislators, rating agencies, governmental practices and the inadequate behavior of supervisory authorities. This crisis can be attributed to a series of invasive factors in the housing and credit markets, to heightened levels of indebtedness, to high risk mortgage products and to inadequate governmental regulation (Roubini, 2009; Steverman, 2008)

3.1.3 Variable Definition

This dissertation is going to study the application of BL in the detection of accounting fraud. The variable in study is the Net Income (NI) of 717 companies, during the period of 2003 to 2012.

According to Damodaran (1997) a company's NI is equal to its earnings minus expenses; earnings originate from the sale of goods or services, and the expenses come from the costs associated with what generates those earnings. The NI calculation is disclosed in the Income Statement (IS).

After literary research, it’s confirmed that the NI variable meets the requirements of Benford's distribution, as pointed out in the studies carried out by authors like Carslaw (1988), Dobsã and Zgela (2011), Forster (2006), Hill (1995) and Thomas (1989).

This variable is accounting data resulting from the combination of other data; and, as it was proven by the investigators, authentic accounting data always follows BD (Carslaw (1988), Dobsã and Zgela (2011), Forster (2006), Hill (1995) and Thomas (1989)). This happens because, according to Hill (1995), most of the accounting data comes from transactions which themselves are the result of combinations of other numbers.

Dobsã and Zgela (2011), applied BL to the NI of 500 companies, Foster (2006), applied BL to earnings and expenses accounts of 159 non-profitable organizations of the Federal district state of Brazil, between the years 2002 and 2003; all the authors confirmed that these data followed BD.

Thomas (1989) analyzed the NI of New Zealand companies and confirmed the results obtained by Carslaw (1988), which was that managers of companies that have positive net income tend to round it off; on the other hand, the companies that report negative net income tend to avoid rounding.

3.2 Methodology

The main methodological procedures to apply to our sample consist of the following steps (Simikin, 2010) :

3.2.1 Selection of the First Digit

Selecting the first digit on the beginning of each NI value. In case some of these values is zero (for example, values smaller than 1), the following digit will have to be considered since, according to BL, it cannot be zero (for instance, if the value of the NI is 0.9, the digit to be considered will be 9).

3.2.2 Calculating the Expected Distribution and Observed Distribution

To create the frequency's distribution, we need to calculate the absolute frequency (Fa) and relative frequency (Fr) for each digit, where Fr is expressed by the following formula:

Fr = Fa/TFa (2)

Where:

Fa = Absolut Frequency

TFa = Absolut Frequency Total

In Excel, the formula used to calculate Fa is the following:

= COUNTIF (Data Range, Criteria)

To calculate the expected distribution, or to recalculate the first digit probability in accordance to BL (these probabilities can be observed in table I), the formula to be used is the following (Simikin, 2010) :

Pdb = Log 10 (1/ 1+d) (3)

Where:

d = Digit

3.2.3 Results Analysis

We intend to verify how the expected and observed distributions relate to each other (Simikin, 2010).

One way of verifying this is to draw both distributions in a single graphic and observe their behavior. BD's graphic should coincide with the observed distribution's graphic (Od). That is to say, the digit 1 on the Od should be the digit with the highest Fr, being the digit 2 the second with higher Fr, and so on (Simikin, 2010).

In order to be able to analyze our results and determine whether the deviations between both probabilities are significant - in case our sample does not correspond to the expected values - we'll have to use a test of hypotheses, in which the null hypothesis (H_0) corresponds to the one where the distribution of the expected probability is equal to the observed distribution.

The Z test allows us to study the level of significance among the differences between both distributions. According to Levin (1987) the level of significance is used to decide whether the obtained sample difference is statistically significant - in accordance to a certain level of confidence which represents the probability where the null hypothesis can be rejected with confidence. The Z value is obtained by using the following formula (Nigrini, 1999) :

$$Z = (|P_o - P_e| - 0.5n) / ((P_e(1 - P_e))/n)^{\wedge}(1/2) \qquad 0.5n < |P_o - P_e| \qquad (4)$$

$$Z = |P_o - P_e| / ((P_e(1 - P_e))/n)^{\wedge}(1/2) \qquad 0.5n \geq |P_o - P_e| \qquad (5)$$

Where:
P_e = Expected Proportion (pe* population)

P_o = Observed Proportion (po* population)

Being that 0,5 n represents the continuity correction term, which is only used when it is smaller than | P_o- P_e |. The level of significance is also α= 0.05, where Z critic equals to 2,131.

All data was analyzed by using the Excel provided by Nigrini, programmed specifically for the application of BL, which includes the previously mentioned methodological procedures. We are also going to carry out the procedures mentioned in his 1999's article (Nigrini, 1999).

Thomas (1989) and Dobsã & Zgela (2011) applied this method to make a separate analysis of the values of companies that made profits and those of the companies that made losses; and confirmed that, in the first case, managers had a tendency to round off the NIs and, in the second, they tended to avoid doing so.

In order to make a similar analysis concerning our sample, we will divide it in positive net income and negative net income and apply the previously described methodology to each one of them. In their turn, these will also be divided in the period before and after 2008, so that we can draw conclusions regarding the 2007/2008 crisis' effects.

3.3 Hypothesis

Taking into account the purpose of this study, the following hypotheses were formulated:

- **H_0:** *It's confirmed that the data follows BD;*
- **H_1:** *It's confirmed that the data does not follow BD;*

Our hypothesis is that we can verify H_0, in other words, that the companies' NI follow BD, since: it describes values within the same phenomenon; it hasn't a defined minimum to the maximum; the values are not attributed numbers and, supposedly, they will not be influenced by psychological factors; thus fulfilling the necessary requirements to be considered a BD.

Additionally, several authors have concluded that the accounting data follows BD (Carslaw, 1998; Thomas, 1989; Zgela & Dobsã, 2011)

If H_1 is verified, then one can assume that the profits are, to a certain degree, a product of psychological influence; and, in that case, the NI could be a consequence of deliberate adjustments (Zgela & Dobsã, 2011).

IV. Result Analysis

4.1 Sample Descriptive Statistics

Our sample's main accounting data can be found in table VI.

Table VI-- Sample's Main Economical Data per Year (2003-2012) (USD Millions)

(USD Millions)	Overall Cash and Cash Equivalent Average	Overall Asset's Average	Overall Liability's Average	Overall Net Income Average	Overall Equity's Average
2003	1211.34	44833.53	44828.05	232.60	2635.97
2004	1338.40	48993.21	49141.60	364.25	2812.16
2005	1331.55	51371.78	51514.89	438.97	3051.81
2006	1390.32	61662.61	61775.51	518.71	3751.53
2007	1902.02	71421.59	71648.08	489.15	4148.14
2008	2349.28	68998.52	69159.22	-80.15	3689.26
2009	2374.47	69972.29	70051.42	265.63	4607.70
2010	3035.96	74632.42	74620.79	427.99	5073.74
2011	3595.61	77103.70	77077.72	440.48	5275.10
2012	4285.43	82171.51	82156.97	429.50	5822.07

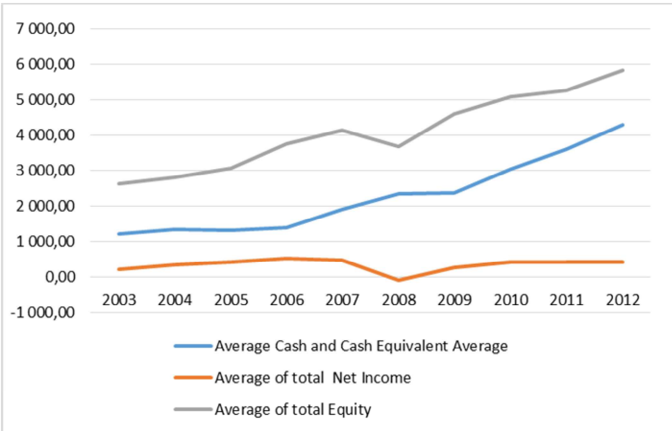
Source: Bloomberg

As shown in figures 1 and 2, one can verify the evolution of our sample's economical data.

Figure 1- Evolution of the Assets and Liabilities Average from Companies in our Sample (2003-2012)

Source: Bloomberg (USD Millions)

Figure 2- Evolution of the CCE net income and the EC's Average from Companies in our Sample.



Source: Bloomberg (USD Millions)

As can be observed in figure 1, during the years 2005 to 2007 there was a quick growth of the assets and liabilities' value in our sample; a decrease in the year of 2008 occurred, which was followed by a growth up to the year of 2012.

In figure 2, it is evident that during the year of 2008 there was a fall in our sample's companies NI value (provoked by the *subprime* crisis), but it was followed by a NI growth on the average of our sample companies up to the year of 2012.

In order to obtain information about the dimension of the companies presented in our sample, we calculated the average number of employees and staff members, where this information was available, which can be seen in table VII.

Table VII- Yearly Average Number of Company Employees

Year	**Average Number of Employees**
2004	19007.00
2005	41852.88
2006	41088.31
2007	23376.38
2008	23129.70
2009	22686.53
2010	21634.09
2011	21675.82
2012	19596.55

Source: Bloomberg

4.2 Positive Net Income

Initially, we applied BL to test the positive NI for the years 2003-2007. The Results can be seen in table VIII and figure 3.

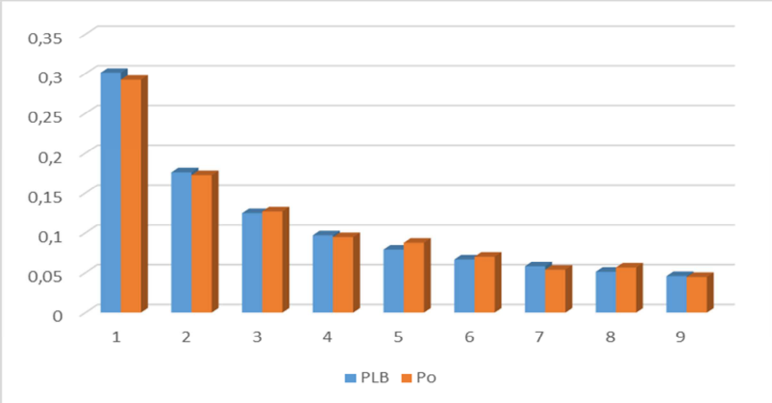
Table VIII- BL for the first digit, analysis of the positive NI (2003-2007)

D	Fe	Fo	PLB	Po	MD	Z Test
0	-	2	-	0.00076	0.00076	
1	796	772	0.30103	0.29209	0.00894	0.9805
2	465	456	0.17609	0.17253	0.00356	0.45497
3	330	336	0.12494	0.12713	0.00219	0.31102
4	256	250	0.09691	0.09459	0.00232	0.37039
5	209	232	0.07918	0.08778	0.0086	160,094
6	177	186	0.06695	0.07037	0.00342	0.66618
7	153	142	0.05799	0.05373	0.00426	0.89653
8	135	149	0.05115	0.05638	0.00523	117,462
9	121	118	0.04576	0.04465	0.00111	0.22686
Total	2,642.00	2,643.00	1.00	1.00		

Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

Subtitle:
D = first digit observed
Fe = expected frequency - number of frequencies according to BL
Fo = observed frequency - number of actual observations
PLB = probability of BL
MD = Deviance Module
Z-test = if the differences are significant, with α=0.5

Figure 3- BL for the first digit, analysis of the positive NI (2003-2007)



Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

In table VIII, we observed that for the digit 1 the number of expected observations was 796; we, however, only obtained 772. Despite the occurrence of this difference, that is not statistically significant.

The digit 5 is the second digit where there are more differences, and according to Zgela & Dobsã, (2011) it shows more often than expected. This can be explained by the influence of psychological factors when creating financial statements; or, specifically, when creating an Income Statement since, according to the authors, it is much more appealing to have a number beginning with 5 than with 4 or even 6.

Finally, we can assert with a confidence degree of 95% that our sample's positive NI follow BL for the years 2003-2007, since no value of the Z test - referring to the differences between the observed and expected frequencies - surpasses Z critic of 2,131, being that the null hypothesis is not rejected.

In figure 3, we can see that the digits that differentiate the most from BD are the digits 1, 5 and 8. Zgela & Dobsã (2011), affirm that in the concept of auditing, digits that appear more often than expected are the ones that deserve more attention and should be investigated; the opposite scenario, that is to say, the digit that appears less frequently, does not deserve so much attention.

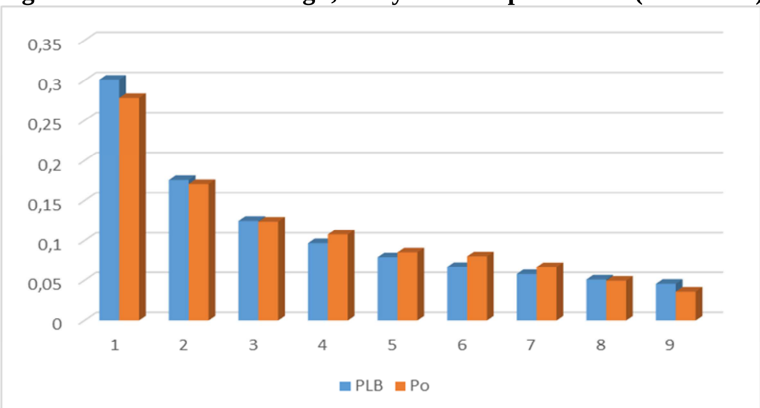
Next, we'll apply BL to test the positive NI for the years 2008-2012. The Results can be seen in table IX and figure 4.

Table IX- BL for the first digit, analysis of the positive NI (2008-2012)

D	Fe	Fo	PLB	Po	MD	Z Test
0	-	2	-	0.00073	0.00073	
1	825	763	0.30103	0.27826	0.02277	257,806*
2	483	469	0.17609	0.17104	0.00505	0.66894
3	343	340	0.12494	0,124	0.00094	0.12025
4	266	296	0.09691	0.10795	0.01104	192,192
5	217	234	0.07918	0.08534	0.00616	115,882
6	184	220	0.06695	0.08023	0.01328	274,554*
7	159	183	0.05799	0.06674	0.00875	191,896
8	140	136	0.05115	0.0496	0.00155	0.32595
9	125	99	0.04576	0.03611	0.00965	237,317*
Total	2,742.00	2,742.00	1.00	1.00		

Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini
*Indicates 95% significantly different from PLB

Figure 4- BL for the first digit, analysis of the positive NI (2008-2012)



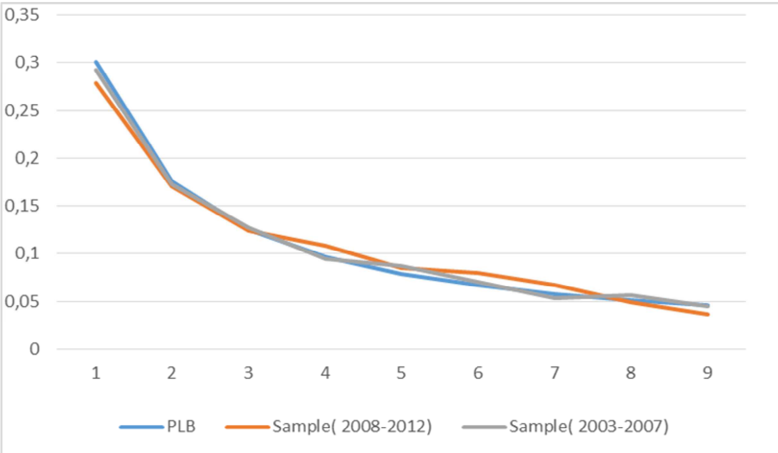
Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

In this case, we can see that there are digits that do not follow BD; and those differences are statistically significant, according to the Z test, since its value is superior to 2,131 (value of the Z critic). The digits that are found in this situation are the digits 1, 6 and 9. These differences also can be explained by the occurrence of the world *subprime* crisis in

2008, which had a great impact in the listed companies' NIs on the financial area stock markets, thus leading to great falls in many companies' NIs.

In order to perform an analysis of the positive NI that follow BD, before and after the year of 2008, we present the figure 5.

Figure 5- BL for the first digit, analysis of the positive NI (2003-2012)



Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

As can be seen, the positive NI for the years of 2003-2007 - with the exception of the digit 5 - follow BD; after 2008, however, our data does not follow BD – with the exception of the digit 2.

4.3 Negative Net Income

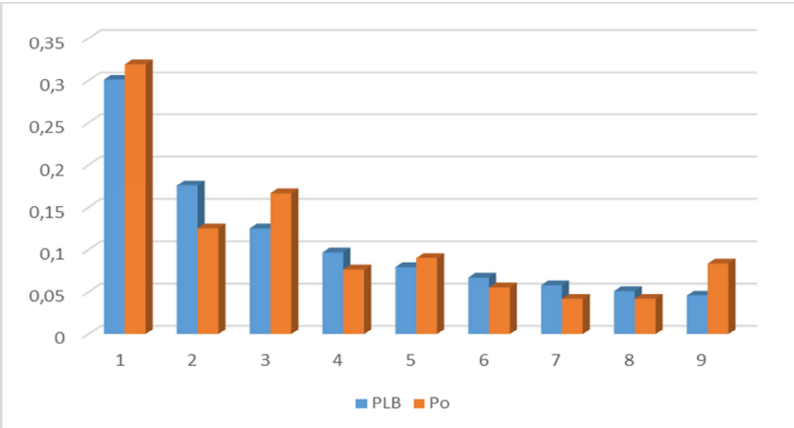
We applied the same methodology to the negative NI data of our sample, having obtained the following results, for the year of 2003- 2007, as shown in table X and figure 6.

Table X- BL for the first digit, analysis of the negative NI (2003-2007)

D	Fe	Fo	PLB	Po	MD	Z Test
0	-	0	-	-	-	
1	43	46	0.30103	0.31944	0.01841	0.3909
2	25	18	0.17609	0,125	0.05109	150,021
3	18	24	0.12494	0.16667	0.04173	138,838
4	14	11	0.09691	0.07639	0.02052	0.69156
5	11	13	0.07918	0.09028	0.0111	0.33883
6	10	8	0.06695	0.05556	0.01139	0.38022
7	8	6	0.05799	0.04167	0.01632	0.6599
8	7	6	0.05115	0.04167	0.00948	0.32756
9	7	12	0.04576	0.08333	0.03757	195,849
Total	143.00	144.00	1.00	1.00		

Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

Figure 6- BL for the first digit, analysis of the negative NI (2003-2007)



Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

We established that the negative NIs of our sample for the year 2003 to 2007 follow BD; since the differences are not statistically significant because the Z test does not surpass the Z-critic of 2,131.

However, it is remarkable that the digit with the biggest difference (in excess) than the one expected is the digit 9. According to several authors, this digit has a higher tendency to appear in negative NI, more often than expected. According to Zgela & Dobsã,

(2011) and Thomas, (1989), this could be the result of the influence of psychological factors in the process of calculating the negative NI, in a specific period, since there is a tendency for not rounding numbers off; and, therefore, we are able to conclude that the difference between the expected and observed values of the digit 9, can be the result of manipulation.

According to Thomas (1989), the observed profits and losses frequencies seem to follow opposite directions. In his study, the author confirms that there's a tendency to round off the NIs when these are positive and to not rounding them off when they are negative. This is why, there are more 9s and more 1s than expected in negative NI.

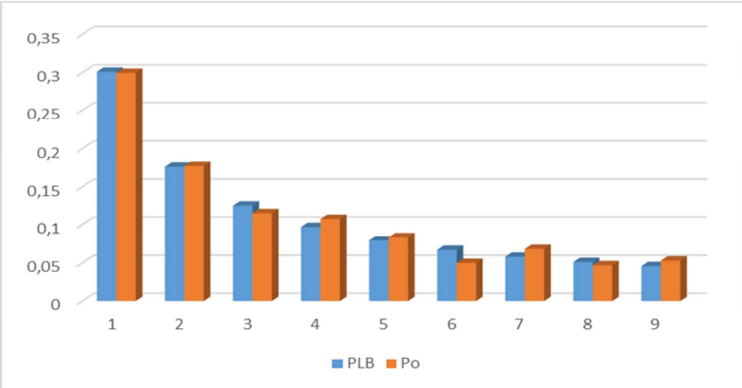
Afterwards, we carried out the same study for the years of 2008-2012; and the obtained results that can be seen in table XI and figure 7.

Table XI- BL for the first digit, analysis of the negative NI (2008-2012)

D	Fe	Fo	PLB	Po	MD	Z Test
0	-	0	-	-	-	
1	199	198	0.30103	0.29955	0.00148	0.04077
2	116	117	0.17609	0,177	0.00091	0.01059
3	83	76	0.12494	0.11498	0.00996	0.71574
4	64	71	0.09691	0.10741	0.0105	0.84704
5	52	55	0.07918	0.08321	0.00403	0.31131
6	44	33	0.06695	0.04992	0.01703	167,326
7	38	45	0.05799	0.06808	0.01009	102,632
8	34	31	0.05115	0.0469	0.00425	0.40815
9	30	35	0.04576	0.05295	0.00719	0.79189
Total	660.00	661.00	1.00	1.00		

Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

Figure 7- BL for the first digit, analysis of the negative NI (2008-2012)

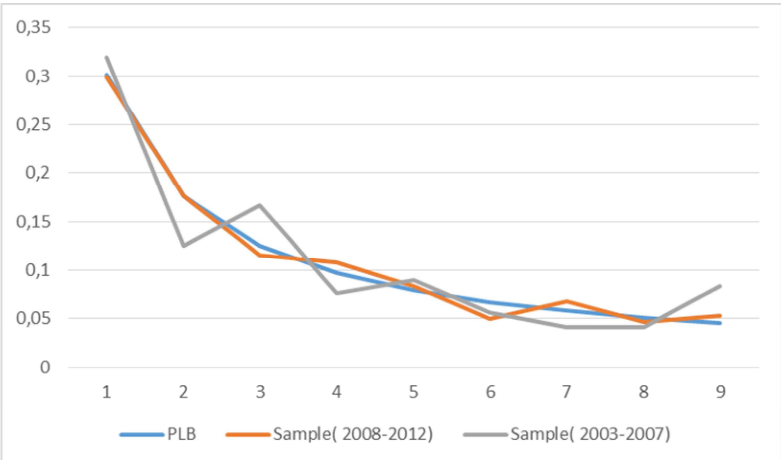


Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

It is clear, in this case, that data follows BD, since all the differences are not statistically significant. Through figure 7, one can visualize that the digit presenting the highest difference is 7. Digit 9 also appears more than was expected, in the years of 2008 to 2012.

Lastly, we compared our sample data for negative NI before and after 2008, as can be seen in figure 8.

Figure 8- BL for the first digit, analysis of the negative NI (2003-2012)



Source: Own Elaboration, through the application of the spreadsheet provided by Nigrini

It's shown that data after 2008 has more tendency to follow BD - with the exception of the digits 7 (with an excess) and digit 6 (with a deficit) - these, however, are not significant differences. As it can be seen, before of the year of 2008, the data does not follow BD and the biggest differences report to the digits 3 and 9 (with an excess) and the digits 7,8 and 2 (with a deficit). However, as previously mentioned, these differences are not significant, since for all of them the value of the Z test is lower than the Z critic. Similarly to the results obtained by Carslaw (1988) and Thomas (1989), our sample also follows BD .

V. Conclusion

5.1 Final Considerations

With the analysis presented in this study we intended to apply Benford's Law to the net income of financial companies listed in several world stock markets; and verify whether our sample followed BD before and after 2008. As it happened to Carslaw (1998) and Thomas (1989), we were able to verify that our sample followed BL for the period in analysis - i.e. during the years of 2003 and 2012 - with the exception of the positive NI after 2008, where the digits 1, 6 and 9 showed significant differences comparing to what was expected, according to BL.

This difference coincides with the *Subprime* crisis which had a worldwide impact up to these days in companies’ results; mainly in those of the financial sector. By observing these differences we are able to assume that profits are, in a certain way, a product of psychological influence, in this case, stated profits after 2008 can be consequence of deliberate adjustments.

Our digit analysis showed that there was a tendency to, in the presence of positive results, rounding numbers off, in this case, the digit 5 before 2008; and, in the presence of negative results, digit 9 tended to be observed in excess, which led us to draw the same conclusion as several authors, such as Carslaw (1988),Thomas (1989) Skousen *et* al., (2004): managers tend to round off the profits and avoid rounding in losses.

Similarly to Thomas (1989), we also verified that the observed frequencies of profits and losses seem to follow opposite directions.

5.2 Criticism and Limitations

In this study we came upon some limitations as, for instance, not being able to conclude on whether the data was fraudulent or not, since deviations do not necessarily imply fraud. Additionally, it was also not possible to obtain evidence either how the fraud occurred, nor to calculate the manipulated value.

5.3 Suggestions for Future Researches

Concerning future researches, it would be interesting to study the applicability of this law in the evaluation of financial fraud in studies focusing on variables of important indicators of a company's economic performance, such as the EBTIDA (Earnings Before Taxes Interest Depreciations and Amortizations).

It would also be important to perform a study focusing only on the year of 2008, year of the world crisis, and not just for companies of the financial sector.

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